SAMPLE

!!CAUTION!!

- **THIS IS ONLY A SAMPLE PAPER**
- **THIS PAPER IS MEANT ONLY FOR PRACTICE**
- **PARTICIPANTS MUST NOT USE THIS SAMPLE AS THE ONLY QUESTIONS TO PREPARE OR TOPICS TO STUDY**
- **ACTUAL COMPETITION WILL BE VARIED AND COVER HIGH SCHOOL PORTION OF MULTIPLE SYLLABI eg. A'LEVEL, IB, NATIONAL CURRICULUM, ARABIC BACCALAUREATE, CBSE, etc...**
- **MATHALON COMMITTEE IS NOT RESPONSIBLE OR ACCOUNTABLE FOR ANY MISCONSTRUED PRACTICE BASED ON THIS SAMPLE**
- **THIS SAMPLE IS NOT MEANT TO BE USED TO PREPARE PARTICIPANTS – JUST TO GIVE AN IDEA OF QUESTION TYPES**
- **IT IS ONLY A REFERENCE AND SHOULD BE USED AS SUCH**

UNIVERSITY
OF WOLLONGONG
IN DUBAI

Faculty of Engineering
and Information Sciences

Section One – MCQ Competition

This section has 22 questions. You MUST answer. Each question carries one (1) mark.

Problem 1

How many ordered pairs of integers (a, b) satisfy all of the following inequalities?

$$a^2 + b^2 < 16$$

$$a^2 + b^2 < 8a$$

$$a^2 + b^2 < 8b$$

A) 12

B) 8

C) 6

D) 16

E) 14

Problem 2

Find the largest number n such that (2004!)! is divisible by ((n!)!)!

A) 6

B) 7

C) 20

D) 16

E) 10

UNIVERSITY OF WOLLONGONG IN DUBAI

Faculty of Engineering and Information Sciences

!!CAUTION!!

THIS IS ONLY A SAMPLE PAPER

Problem 3

Let x be a real number such that $x^3 + 4x = 8$. Determine the value of

$$x^7 + 64x^2$$

A) 168

B) 878

C) 208

D) 128

E) 108

Problem 4

Compute:

$$\left\lfloor \frac{2005^3}{2003 \cdot 2004} - \frac{2003^3}{2004 \cdot 2005} \right\rfloor$$

Where $\lfloor x \rfloor$ denotes the greatest integer less than x.

A) 7

B) 9

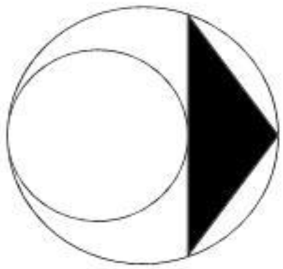
C) 10

D) 11

E) 8

Problem 5

In the diagram below, the outer circle has radius 3, and the inner circle has radius 2. What is the area of shaded region?



A) $2\sqrt{2}$

B) $4\sqrt{2}$

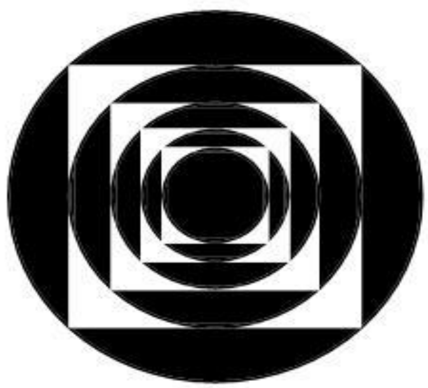
C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{3}}{2}$

E) $2\sqrt{3}$

Problem 6

We inscribe a square in a circle of radius 1 and shade the region between them. Then we inscribe another circle in the square and another square in the new circle and shade the region between the new circle and square. After we have repeated this process infinitely many times, what is the area of the shaded region?



A) $\frac{\pi}{4} - 4$

B) $2\pi - 4$

C) $2\pi + 1$

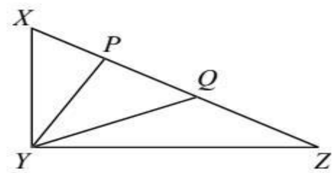
D) $\pi - \frac{1}{4}$

E) $\pi + \frac{1}{4}$

Problem 7

Right triangle XYZ has right angle at Y and $XY = 228$, $YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of

$$(PY + YZ)(QY + XY)$$



A) 228404

B) 9887244

C) 1370736

D) 76542964

E) 1370736

Problem 8

A parallelogram has 3 of its vertices at (1, 2), (3, 8), and (4, 1). Compute the sum of the possible x-coordinates for the 4th vertex.

A) 4

B) 6

C) 12

D) 8

E) 11

Problem 9

Find the area of the region of the xy-plane defined by the inequality

$$|x| + |y| + |x + y| \leq 1$$

A) 3/4

B) 1/4

C) 3/8

D) 13/8

E) 5/4

Problem 10

In the Championship World Series of the future, Rice and Stanford play three games or until one team wins two games. In each game, both teams have a $\frac{1}{3}$ of winning, and there is a $\frac{1}{3}$ chance that they will tie. What is the probability that Rice wins the championship?

A) 7/26

B) 7/33

C) 1/9

D) 9/13

E) 8/23

Problem 11

There are 1000 rooms in a row along a long corridor. Initially the first room contains 1000 people and the remaining rooms are empty. Each minute, the following happens: for each room containing more than one person, someone in that room decides it is too crowded and moves to the next room. All these movements are simultaneous (so nobody moves more than once within a minute). After one hour, how many different rooms will have people in them?

A) 91

B) 500

C) 31

D) 41

E) 600

Problem 12

What is the largest whole number that is equal to the product of its digits?

A) 10

B) 9

C) 19

D) 900

E) 90

Problem 13

Principal Skinner is thinking of two integers m and n and bets Superintendent Chalmers that he will not be able to determine these integers with a single piece of information. Chalmers asks Skinner the numerical value of

$$mn + 13m + 13n - m^2 - n^2$$

From the value of this expression alone, he miraculously determines both m and n. What is the value of the above expression?

A) 26

B) 169

C) 39

D) 139

E) 133

Problem 14

Jerry is bored one day, so he makes an array of Cocoa pebbles. He makes 8 equal rows with the pebbles remaining in a box. When Kramer drops by and eats one, Jerry yells at him until Kramer realizes he can make 9 equal rows with the remaining pebbles. After Kramer eats another, he finds he can make 10 equal rows with the remaining pebbles. Find the smallest number of pebbles that were in the box in the beginning.

A) 110

B) 352

C) 113

D) 310

E) 320

Problem 15

Let $f(a,b) = \frac{1}{a+b}$ when $a + b \neq 0$. Suppose that x, y, z are distinct integers such that

$$x + y + z = 2015$$

and $f(f(x,y),z) = f(x,f(y,z))$ (where both sides of the equation exist and are well defined). Compute y.

A) -2015

B) 2015

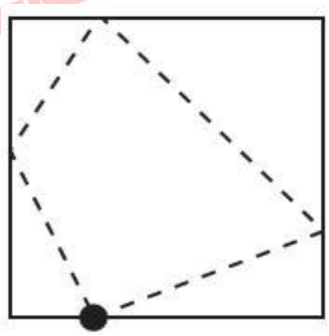
C) 2045

D) -2045

E) 2030

Problem 16

John stands against one wall of a square room with walls of length 4 meters each. He kicks a frictionless, perfectly elastic ball in such a way that it bounces off the three other walls once each and returns to him (diagram not geometrically accurate). How many meters does the ball travel?



A) $\frac{\sqrt{2}}{4}$

B) $2\sqrt{2}$

C) $\sqrt{2}$

D) $4\sqrt{2}$

E) $8\sqrt{2}$

Problem 17

P is inside rectangle ABCD. PA = 2, PB = 3, and P C = 10. Find PD.

A) $\frac{\sqrt{75}}{2}$

B) $2\sqrt{91}$

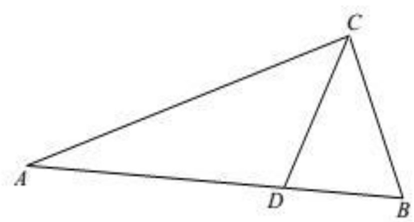
C) $\sqrt{83}$

D) $\sqrt{21}$

E) $\sqrt{95}$

Problem 18

AC is 2004. CD bisects angle C. If the perimeter of ABC is 6012, find $\frac{AC \cdot BC}{AD \cdot BD}$



A) 8

B) 7

C) 5

D) 4

E) 9

Problem 19

A horse stands at the corner of a chessboard (above), a white square. With each jump, the horse can move either two squares horizontally and one vertically or two vertically and one horizontally (like a knight moves). The horse earns two carrots every time it lands on a black square, but it must pay a carrot in rent to rabbit who owns the chessboard for every move it makes. When the horse reaches the square on which it began, it can leave. What is the maximum number of carrots the horse can earn without touching any square more than twice?

A) 5

B) 0

C) 7

D) 3

E) 1

Problem 20

Given that $x^2 - 3x + 1 = 0$, find

$$x^9 + x^7 + x^{-9} + x^{-7}$$

A) 3455

B) 6621

C) 7771

D) 1122

E) 1878

Problem 21

Find $sinx - cosx$ if

$$sin2x = \frac{2002}{2003} \text{ and } \frac{5\pi}{4} < x < \frac{9\pi}{4}$$

A) $\frac{-1}{\sqrt{2003}}$

B) $\sqrt{2002}$

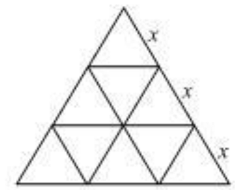
C) $\frac{\sqrt{2003}}{2002}$

D) $\frac{-1}{\sqrt{2002}}$

E) $\sqrt{2003}$

Problem 22

Each of the small equilateral triangles (9 total) have side length x and is randomly coloured red or blue. What is the probability that there will be an equilateral triangle of side length 2x or 3x that is entirely red or entirely blue?



A) $\frac{5}{128}$

B) $\frac{9}{5}$

C) $\frac{31}{512}$

D) $\frac{5}{9}$

E) $\frac{1}{2}$

END of SECTION ONE

Section Two – Tie Breaker

This section has two (2) questions. You MUST answer ALL parts showing ALL steps clearly in spaces provided below. You MUST answer in the *space provided*. *Show* *all workings*. *Marks for this section are 18 (9 marks each)*

Problem 1

If a, b, c are real numbers such that

$$a + b + c = -1$$

$$a^2 + b^2 + c^2 = 17$$

$$a^3 + b^3 + c^3 = 11,$$

find abc.

Problem 2

Which is bigger, the coefficient of x^{1000} in $(1 - x^2 + x^5)^{2005}$ or the coefficient of x^{1000} in $(1 + x^2 - x^5)^{2005}$? Justify your answer.

!!CAUTION!!

THIS IS ONLY A SAMPLE PAPER